Analysis of $\Lambda_b \to \Lambda_c$ weak decays in Heavy Quark Effective Theory

Jong-Phil Lee*, Chun Liu† and H. S. Song‡

Department of Physics and Center for Theoretical Physcis,

Seoul National University, Seoul, 151-742, Korea

Abstract

The $\Lambda_b \to \Lambda_c$ semileptonic decay is analyzed in the framework of heavy quark effective theory to the order of $1/m_c$ and $1/m_b$. The QCD sum rule and large N_c predictions to the decay form factors are applied. It argues that the subleading baryonic Isgur-Wise function in the large N_c limit vanishes. The decay rates, distributions and asymmetry parameters are calculated numerically. Some of the nonleptonic decay modes are discussed in the end.

*jplee@phya.snu.ac.kr

†liuc@ctp.snu.ac.kr

‡hssong@physs.snu.ac.kr

I. INTRODUCTION

The weak decays of heavy baryons provide testing ground for the Standard Model. They reveal some important features of the physics of heavy quarks. From the study of the heavy quark physics, some important parameters of the Standard Model, for instance, the Cabbibo-Kobayashi-Maskawa (CKM) matrix element V_{cb} can be extracted by comparing experiments with theoretical calculations from the decay mode $\Lambda_b \to \Lambda_c l\bar{\nu}$. The main difficulties in the Standard Model calculations, however, are due to the poor understanding of the nonperturbative aspects of the strong interactions (QCD).

For the heavy hadrons containing a single heavy quark, an effective theory of QCD based on the heavy quark symmetry in the heavy quark limit [1], the so-called heavy quark effective theory (HQET), has been proposed [2]. The classification of the weak decay form factors of heavy baryons has been simplified greatly in HQET [3]. At the leading order of heavy quark expansion, only one universal form factor, the Isgur-Wise function, is required to describe the $\Lambda_b \to \Lambda_c$ semileptonic decay. To the order of $1/m_Q$ [4], one more universal function and one mass parameter are introduced [5]. However, the heavy quark symmetry itself has no power to give information about the details of the universal form factors and the mass parameter. For a complete analysis to the heavy baryons, we still need to employ some other nonperturbative methods. Interesting results about the heavy baryon weak decay form factors have been obtained by various nonperturbative methods. They are QCD sum rules [6,7], large N_c limit [8], lattice simulation [9], dispersion relation and analyticity [10], and quark models [11].

In this paper, we apply the results of QCD sum rules and large N_c limit to analyze in detail the weak decays of $\Lambda_b \to \Lambda_c$ to the order of $1/m_c$ and $1/m_b$. The analysis is useful to experiments in the near future. In Sec.2, $\Lambda_b \to \Lambda_c$ semileptonic decay form factors are discussed. While there is no large N_c calculation for the universal form factor appeared in $1/m_Q$ corrections, we argue that it is zero in the large N_c limit. In Sec.3, the numerical results for the decay rates, distributions and various angular asymmetry parameters are

calculated. In Sec.4, several non-leptonic decay modes of Λ_b are discussed. We summarize the results in Sec.5.

II. FORM FACTORS

The hadronic matrix element of the weak current appeared in the effective Hamiltonian for $\Lambda_b \to \Lambda_c$ is parameterized generally by six form factors F_i and G_i $(i = 1, 2, 3)$,

$$\begin{aligned} \langle \Lambda_c(v')|\bar{c}\gamma^\mu(1-\gamma^5)b|\Lambda_b(v)\rangle &= \bar{u}_{\Lambda_c}(v')(F_1\gamma^\mu + F_2 v^\mu + F_3 v'^\mu)u_{\Lambda_b}(v) \\ &\quad -\bar{u}_{\Lambda_c}(v')(G_1\gamma^\mu + G_2 v^\mu + G_3 v'^\mu)\gamma^5 u_{\Lambda_b}(v) \ , \end{aligned} \tag{1}$$

where v and v' denote the four-velocities of Λ_b and Λ_c respectively. Within the framework of HQET, the classification of the form factors is simplified very much. To the order of both $1/m_c$ and $1/m_b$, they are expressed as

$$\begin{aligned} F_1 &= C(\mu)\xi(y) + (\frac{\bar{\Lambda}}{2m_c} + \frac{\bar{\Lambda}}{2m_b})[2\chi(y) + \xi(y)] \ , \\ G_1 &= C(\mu)\xi(y) + (\frac{\bar{\Lambda}}{2m_c} + \frac{\bar{\Lambda}}{2m_b})[2\chi(y) + \frac{y-1}{y+1}\xi(y)] \ , \\ F_2 &= G_2 = -\frac{\bar{\Lambda}}{m_c(y+1)}\xi(y) \ , \\ F_3 &= -G_3 = -\frac{\bar{\Lambda}}{m_b(y+1)}\xi(y) \end{aligned} \tag{2}$$

with the perturbative QCD coefficient in the leading logarithmic approximation

$$C(\mu) = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{\frac{6}{25}} \left[\frac{\alpha_s(m_c)}{\alpha_s(\mu)}\right]^{a_L(y)} \ , \tag{3}$$

where

$$a_L(y) = \frac{8}{27}[yr(y) - 1] \ , \quad r(y) = \frac{1}{\sqrt{y^2-1}}\ln(y + \sqrt{y^2-1}). \tag{4}$$

ξ and χ are the so-called leading and subleading Isgur-Wise function respectively. And the mass parameter $\bar{\Lambda}$ is defined as follows

$$\bar{\Lambda} = m_{\Lambda_Q} - m_Q \ . \tag{5}$$

By QCD sum rules, ξ, χ and $\bar{\Lambda}$ have been obtained [7]. QCD sum rule is regarded as a nonperturbative method rooted in QCD itself [12]. In a linear approximation, the leading Isgur-Wise function is fit as

$$\xi(y) = 1 - \rho^2(y-1)\ , \qquad \rho^2 = 0.55 \pm 0.15\ . \tag{6}$$

On the other hand, the subleading Isgur-Wise function is negligibly small,

$$\chi(y) \simeq O(10^{-2})\ . \tag{7}$$

And the parameter $\bar{\Lambda}$ is determined to be

$$\bar{\Lambda} = 0.79 \pm 0.05 \text{ GeV}\ . \tag{8}$$

It is interesting to compare above QCD sum rule results with that of large N_c. Large N_c limit is one of the most important and model-independent method of nonperturbative QCD in spite of the realistic $N_c = 3$ [13]. HQET in this limit, for the heavy baryon case, is often believed to be the heavy quark Skyrme model [14]. The leading Isgur-Wise function is predicted as [8]

$$\xi(y) = 0.99 \exp[-1.3(y-1)]\ . \tag{9}$$

The slope of this Isgur-Wise function is steeper than that of the sum rule. In the large N_c limit, the parameter $\bar{\Lambda}$ equals to the proton mass [15]. In the following analysis, we take it as $\bar{\Lambda} \simeq 0.87$ GeV [15]. This result is in agreement with that obtained by QCD sum rules. However, there is no Skyrme model calculation for the subleading Isgur-Wise function.

We will assume that the subleading Isgur-Wise function is negligible in the Skyrme model analysis. In the following, we argue that this assumption is true in the large N_c limit. The subleading Isgur-Wise function $\chi(y)$ is defined by

$$\begin{aligned} &\langle \Lambda_c(v')|\mathrm{T}\bar{h}_{v'}^{(c)}\Gamma h_v^{(b)} i\int d^4x \frac{1}{2m_Q}\bar{h}_v^{(Q)}(x)(iD)^2 h_v^{(Q)}(x)|\Lambda_b(v)\rangle \\ &= \frac{\bar{\Lambda}}{m_Q}\chi(y)\bar{u}_{\Lambda_c}(v')\Gamma u_{\Lambda_b}(v)\ , \end{aligned} \tag{10}$$

(11)

where $h_v^{(Q)}$ denotes the heavy quark field defined in the HQET with velocity v, and Γ is some gamma matrix. $\chi(y)$ just measures the amplitude of the brown muck transfer through a strong interaction, described by above matrix element, from the heavy quark which has a velocity change from v to v' due to the weak decay. In the Hartree-Fock picture of large N_c HQET, heavy baryon has $N_c - 1$ light quarks. Any $v \neq v'$ $\;(y \neq 1)$ transition in fact is suppressed when N_c is large, because that involves changing the momenta of all the light quarks inside the baryon. In the limit $N_c \to \infty$, we expect $\chi(y \neq 1) = 0$. Furthermore, it is well-known that $\chi(1) = 0$ due to Luke theorem [4]. Therefore, we get that $\chi(y)$ vanishes in the large N_c limit. Although the above argument makes our assumption for $\chi(y)$ reasonable, it should be noted that there is still a subtle point which distinguishes the Skyrme model from the large N_c limit. The point is that for heavy baryon weak decay form factors, the Skyrme model result is not exactly identical to that of large N_c. Consider the leading Isgur-Wise function, our large N_c argument for $\chi(y)$ also applies to $\xi(y)$, that is $\xi(y \neq 1) = 0$. Because $\xi(1) = 1$, we expect that the leading Isgur-Wise function is δ-function like, $\xi(y) \sim \delta(y-1)$ in the large N_c limit. However, this result in principle agrees with that of Skyrme model [8] $\xi(y) \sim \exp[-N_c^{3/2}(y-1)]$, if N_c is taken to be ∞. The so called Skyrme model result Eq.(9) can be obtained by taking $N_c = 3$. Therefore, $\chi(y) = 0$ can be understood as the result of large N_c limit for the Skyrme model.

III. DECAY RATES, DISTRIBUTIONS AND ASYMMETRY PARAMETERS

With the knowledge of the form factors from QCD sum rule and large N_c limit, we can calculate the rates, distributions and various asymmetry parameters for the $\Lambda_b \to \Lambda_c$ semileptonic decay. The standard expressions for these ovservables are given in Ref. [16] in terms of helicity amplitudes. The process $\Lambda_b \to \Lambda_c l \bar{\nu}$ is considered as a two-successive decay $\Lambda_b \to \Lambda_c + W_{\rm off-shell}$, $W_{\rm off-shell} \to l + \nu$. Let $\epsilon^{\mu}_{\lambda_W}$ be the polarization vector of $W_{\rm off-shell}$, where λ_W denotes the helicity state. Longitudinal state corresponds to $\lambda_W = 0$, whereas transverse state, $\lambda_W = \pm 1$. The helicity amplitudes are defined by

$$H^{V(A)}_{\pm\frac{1}{2}\lambda_W} = \epsilon^{\mu}_{\lambda_W}\langle\Lambda_c(v';\pm\frac{1}{2})|J^{V(A)}_{\mu}|\Lambda_b(v)\rangle \ , \tag{12}$$

where $J^{V(A)}$ stands for the vector(axial vector) current, and $\pm\frac{1}{2}$ in the subscript is the helicity of the daughter baryon Λ_c. They can be expressed by the form factors,

$$\begin{aligned}
\sqrt{q^2}H^{V,A}_{\frac{1}{2}0} &= \sqrt{2M_{\Lambda_b}M_{\Lambda_c}(y\mp 1)}\{(M_{\Lambda_b}\pm M_{\Lambda_c})(F_1,G_1) \\
&\qquad \pm M_{\Lambda_c}(y\pm 1)(F_2,G_2)\pm M_{\Lambda_b}(y\pm 1)(F_3,G_3)\} \ , \\
H^{V,A}_{\frac{1}{2}1} &= -2\sqrt{M_{\Lambda_b}M_{\Lambda_c}(y\mp 1)}(F_1,G_1) \ ,
\end{aligned} \tag{13}$$

where the upper(lower) sign is for the vector(axial vector) current. With the notation of the total helicity amplitude $H_{\pm\frac{1}{2}\lambda_W} = H^V_{\pm\frac{1}{2}\lambda_W} - H^A_{\pm\frac{1}{2}\lambda_W}$, and the parity relation $H^{V(A)}_{\mp\frac{1}{2}-\lambda_W} = (-)H^{V(A)}_{\pm\frac{1}{2}\lambda_W}$, the differential decay rate can be expressed as

$$\begin{aligned}
\frac{d\Gamma}{dy\ d\cos\theta} &= \frac{G_F^2}{(2\pi)^3}|V_{cb}|^2 q^2\sqrt{y^2-1}\frac{M^2_{\Lambda_c}}{M_{12\Lambda_b}}[\frac{3}{8}(1+\cos\theta)^2|H_{\frac{1}{2}1}|^2 \\
&\quad +\frac{3}{8}(1-\cos\theta)^2|H_{-\frac{1}{2}-1}|^2+\frac{3}{4}\sin^2\theta(|H_{\frac{1}{2}0}|^2+|H_{-\frac{1}{2}0}|^2)],
\end{aligned} \tag{14}$$

where θ is the angle between P_{Λ_c} and p_l measured in the $W_{\text{off}-\text{shell}}$ rest frame. The y distribution of the decay rate is obtained by the integration over $\cos\theta$,

$$\begin{aligned}
\frac{d\Gamma}{dy} &= \frac{G_F^2}{(2\pi)^3}|V_{cb}|^2 q^2\sqrt{y^2-1}\frac{M^2_{\Lambda_c}}{12M_{\Lambda_b}}\left[|H_{\frac{1}{2}1}|^2+|H_{-\frac{1}{2}-1}|^2+|H_{\frac{1}{2}0}|^2+|H_{-\frac{1}{2}0}|^2\right] \\
&= \frac{d\Gamma_{T+}}{dy}+\frac{d\Gamma_{T-}}{dy}+\frac{d\Gamma_{L+}}{dy}+\frac{d\Gamma_{L-}}{dy} \ ,
\end{aligned} \tag{15}$$

where $T_\pm$, $L_\pm$ are defined as the transerve and longitudinal contribution to the decay rate with $\pm$ final baryon helicity, respectively.

The numerical results are obtained by inputting the form factors discussed in last section. We have taken $m_c = 1.44$ GeV, $m_b = 4.83$ GeV, $\mu = 0.47$ GeV and $|V_{cb}| = 0.04$. The partial decay distributions are plotted as a function of y in Fig.1 and Fig.2 for both QCD sum rule and large N_c predictions. It is easy to see the dominance of $\frac{d\Gamma_{T-}}{dy}$ and $\frac{d\Gamma_{L-}}{dy}$ over other plus helicity components. As discussed in Ref. [17], this is due to the left-handed V-A current. From Fig.2, we can see that the discrepancy of different model gets larger as y goes larger.

As a result of the fact that the slope of the Isgur-Wise function of large N_c is steeper than that of QCD sum rule, the decay distributions on y predicted by large N_c are smaller than that of QCD sum rule explicitly when $y \gtrsim 1.1$.

It is experimentally useful to calculate the lepton energy distribution. We obtain [1],

$$
\begin{aligned}
\frac{d\Gamma}{dE_l} &= \frac{G_F^2}{(2\pi)^3}|V_{cb}|^2 \frac{M_{\Lambda_c}^2}{8} \int_{y_{min}(E_l)}^{y_{max}} dy(y_{max}-y)\Big[(1+\cos\theta)^2|H_{\frac{1}{2}1}|^2 \\
&\quad +(1-\cos\theta)^2|H_{-\frac{1}{2}-1}|^2 + 2\sin^2\theta(|H_{\frac{1}{2}0}|^2+|H_{-\frac{1}{2}0}|^2)\Big] \\
&\equiv \frac{d\Gamma_{T_+}}{dE_l} + \frac{d\Gamma_{T_-}}{dE_l} + \frac{d\Gamma_{L_+}}{dE_l} + \frac{d\Gamma_{L_-}}{dE_l}
\end{aligned} \tag{16}
$$

where

$$
\begin{aligned}
\cos\theta &= \frac{E_l^{max} - 2E_l + M_{\Lambda_c}(y_{max}-y)}{M_{\Lambda_c}\sqrt{y^2-1}} \\
E_l^{max} &= \frac{M_{\Lambda_b}^2 - M_{\Lambda_c}^2}{2M_{\Lambda_b}} \\
y_{max} &= \frac{M_{\Lambda_b}^2 + M_{\Lambda_c}^2}{2M_{\Lambda_b}M_{\Lambda_c}} \\
y_{min}(E_l) &= y_{max} - 2\frac{E_l}{M_{\Lambda_c}}\frac{E_l^{max}-E_l}{M_{\Lambda_b}-2E_l}.
\end{aligned} \tag{17}
$$

The lepton energy spectrums of the decay rates are given in Fig.3 and Fig.4 for the QCD sum rule and large N_c Isgur-Wise function. As in the case of y distribution, the helicity minus components dominate the plus ones. And the decay distributions on E_l by large N_c are always smaller than that by QCD sum rule. The decay rates are obtained from Eq.(15) by integrating over y, or from Eq.(16) by integrating over E_l. The numerical results for the partial decay rates into given helicity states are listed in Table 1, where the quark model results [17] are also listed for comparison. The total decay rate is obtained by summing them up,

$$
\Gamma = 6.17\times 10^{-14}\ \text{GeV}\ ,\ \text{Br.}(\Lambda_b \to \Lambda_c l\bar{\nu}) = 11.5\% \times \Big(\frac{\tau(\Lambda_b)}{1.23\times 10^{-12}\ \text{sec}}\Big) \tag{18}
$$

for QCD sum rule linear fitting, and

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[1]Our results for $\frac{d\Gamma_{L_\pm}}{dE_l}$ are different from that given in Ref. [11].

$$\Gamma = 4.51 \times 10^{-14} \text{ GeV} \ , \ \text{Br.}(\Lambda_b \to \Lambda_c l\bar{\nu}) = 8.43\% \times \Big(\frac{\tau(\Lambda_b)}{1.23 \times 10^{-12} \text{ sec}}\Big) \tag{19}$$

for large N_c approximation. The quark model results given in Ref. [17] are $\Gamma = 4.28 \times 10^{-14}$ GeV and Br. $= 7.99\%$ for $\tau(\Lambda_b) = 1.23 \times 10^{-12}$ sec. The QCD sum rule predicts larger decay branching ratio than large N_c model, as we have expected. We also see that both QCD sum rule and large N_c predicts larger results for the decay than quark model of Ref. [17]. Up to the leading order, we have $\Gamma = 5.52 \times 10^{-14}$ GeV for QCD sum rule and $\Gamma = 4.00 \times 10^{-14}$ GeV for large N_c limit. It means that $1/m_c$ and $1/m_b$ corrections yield about 11% enhancement for the total decay rate.

Now let's turn to the various asymmetry paremeters. The polarization effects in the process $\Lambda_b \to \Lambda_c$ are revealed in various angular distributions. First, from Eq.(14), the polar angle distribution is

$$\frac{d\Gamma}{dy\ d\cos\theta} \propto 1 + 2\alpha' \cos\theta + \alpha'' cos^2\theta, \tag{20}$$

where α' and α'' are asymmetry parameters which can be expressed as

$$\alpha' = \frac{|H_{1/2\ 1}|^2 - |H_{-1/2-1}|^2}{|H_{1/2\ 1}|^2 + |H_{-1/2-1}|^2 + 2(|H_{1/2\ 0}|^2 + |H_{-1/2\ 0}|^2)} \tag{21}$$

$$\alpha'' = \frac{|H_{1/2\ 1}|^2 + |H_{-1/2-1}|^2 - 2(|H_{1/2\ 0}|^2 + |H_{-1/2\ 0}|^2)}{|H_{1/2\ 1}|^2 + |H_{-1/2-1}|^2 + 2(|H_{1/2\ 0}|^2 + |H_{-1/2\ 0}|^2)} \ . \tag{22}$$

There are other asymmetry parameters if the successive hadronic cascade decay $\Lambda_c \to a + b$ where a and b are some hadrons, are considered. Two more angles are involved, Θ_Λ and χ. Θ_Λ is the angle beween Λ_c's momentum in Λ_b rest frame and a's momentum in Λ_c rest frame assuming $J_a = \frac{1}{2}$ and $J_b = 0$. χ is the relative azimuthal angle between the decay planes defined by $l,\ \nu$ and $a,\ b$. Θ_Λ and χ distributions of the decay are [16]

$$\frac{d\Gamma}{dy\ d\cos\Theta_\Lambda} \propto 1 + \alpha\alpha_\Lambda \cos\Theta_\Lambda \quad \text{and} \quad \frac{d\Gamma}{dy\ d\chi} \propto 1 - \frac{3\pi^2}{32\sqrt{2}}\gamma\alpha_\Lambda \cos\chi, \tag{23}$$

where α_Λ is the asymmetry parameter in the Λ_c hadronic decay. In this case, the related asymmetry parametes, α and γ are given by

$$\alpha = \frac{|H_{1/2\ 1}|^2 - |H_{-1/2-1}|^2 + |H_{1/2\ 0}|^2 - |H_{-1/2\ 0}|^2}{|H_{1/2\ 1}|^2 + |H_{-1/2-1}|^2 + |H_{1/2\ 0}|^2 + |H_{-1/2\ 0}|^2}, \tag{24}$$

$$\gamma = \frac{2\mathrm{Re}(\mathrm{H}_{-1/2\ 0}\mathrm{H}^*_{1/2\ 1} + \mathrm{H}_{1/2\ 0}\mathrm{H}^*_{-1/2-1})}{|H_{1/2\ 1}|^2 + |H_{-1/2-1}|^2 + |H_{1/2\ 0}|^2 + |H_{-1/2\ 0}|^2}. \tag{25}$$

When the Λ_b polarization is further considered, additional asymmetry parameters can be introduced. The new decay angles related are Θ_P and χ_P. Θ_P is the angle between Λ_b polarization and Λ_c momentum, and χ_P is the azimuthal angle between the plane of Λ_b polarization, Λ_c momentum and that of a, b's momenta. The decay distributions are [16]

$$\frac{d\Gamma_{pol}}{dy\ d\cos\Theta_P} \propto 1 - \alpha_P P \cos\Theta_P, \quad \text{and} \quad \frac{d\Gamma_{pol}}{dy\ d\chi_P} \propto 1 - \frac{\pi^2}{16} P \alpha_\Lambda \gamma_P \cos\chi_P, \tag{26}$$

where P is the degree of polarization of Λ_b. The asymmetry parameters α_P and γ_P are

$$\alpha_P = \frac{|H_{1/2\ 1}|^2 - |H_{-1/2-1}|^2 - |H_{1/2\ 0}|^2 + |H_{-1/2\ 0}|^2}{|H_{1/2\ 1}|^2 + |H_{-1/2-1}|^2 + |H_{1/2\ 0}|^2 + |H_{-1/2\ 0}|^2}, \tag{27}$$

$$\gamma_P = \frac{2\mathrm{Re}(\mathrm{H}_{1/2\ 0}\mathrm{H}^*_{-1/2\ 0})}{|H_{1/2\ 1}|^2 + |H_{-1/2-1}|^2 + |H_{1/2\ 0}|^2 + |H_{-1/2\ 0}|^2}\ . \tag{28}$$

These asymmetry parameters are functions of y. On averaging over y, the numerators and denominators are integrated separately with proper weight, $(y_{max} - y)\sqrt{y^2-1}$.

Our numerical results on the mean values of the asymmetry parameters are listed in Table 2 where both QCD sum rule and large N_c results are given. Note that these results include $1/m_c$ and $1/m_b$ corrections. The quark model results [17] are also listed for comparison. Note that all the results include $1/m_c$ and $1/m_b$ corrections.

IV. NONLEPTONIC DECAYS

In this section, we will consider the two-body nonleptonic decay modes $\Lambda_b \to \Lambda_c \pi(\rho)$ and $\Lambda_b \to \Lambda_c K^{(*)}$. All these decays involve external W-emission diagrams which can be analyzed by using the factorization approximation [18,19]. $\Lambda_b \to \Lambda_c \pi(\rho)$ also get contributions from internal W-emission which, however, is non-factorizable and is difficult to calculate reliably.

We will simply neglect this contribution in the analysis. Penguin diagrams do not contribute to these decays. There are contributions from W-exchange diagrams in both $\Lambda_b \to \Lambda_c \pi(\rho)$ and $\Lambda_b \to \Lambda_c K^{(*)}$ channels. Because such diagrams are suppressed by the possibility of the two valence quark lines meeting in the region of $1/M_W$, they are neglected. This is also justified by a detailed quark model analysis for b-baryon nonleptonic decays [20]. In short, the decays will be analyzed by using factorization assumption which is expected to be good except for the $\Lambda_c \pi(\rho)$ channels. In a recent study [21], the non-factorizable effect in decay $\Lambda_b \to \Lambda_c \pi$ has been estimated. The total non-factorizable contribution is about 30% of the factorizable one. Although it is indeed sizable, the factorizable effect is still dominant.

After factorization, the amplitude of the process can be expressed by the product of two matrix elements to which the form factors given in Sec.II can be applied just as in the case of the semileptonic decay. With the definition of the $\Lambda_b \to \Lambda_c$ matrix element Eq.(1), the widths of the decays into pseudoscalar meson(P) and vector meson(V) can be easily calculated:

$$\Gamma(P) = \frac{G_F^2}{2\pi} |V_{cb} V_{ij}|^2 f_P^2 a_1^2 M_{\Lambda_b}^3 \frac{r^2 \kappa_P}{(1-\kappa_P^2)^2} [|A|^2 + \kappa_P^2 |B|^2] \ , \tag{29}$$

where

$$\begin{aligned} A &= (1-r)\Big[F_1 + \frac{1+r}{2}(F_2 + \frac{F_3}{r})\Big] \ , \\ B &= (1+r)\Big[G_1 - \frac{1-r}{2}(G_2 + \frac{G_3}{r})\Big] \ , \\ \kappa_{P(V)} &= \sqrt{\frac{(1-r)^2 - t_{P(V)}^2}{(1+r)^2 - t_{P(V)}^2}} \ , \end{aligned} \tag{30}$$

and

$$\Gamma(V) = \frac{G_F^2}{4\pi} |V_{cb} V_{ij}|^2 a_1^2 f_V^2 M_{\Lambda_b}^3 \frac{r^2 \kappa_V}{(1-\kappa_V^2)^2} [2t_V^2(\kappa_V^2 |F_1|^2 + |G_1|^2) + \kappa_V^2 |C|^2 + |D|^2] \ , \tag{31}$$

where

$$\begin{aligned} C &= (1+r)F_1 + \frac{2r}{1-\kappa_V^2}(F_2 + \frac{F_3}{r}) \ , \\ D &= (1-r)G_1 - \frac{2r\kappa_V^2}{1-\kappa_V^2}(G_2 + \frac{G_3}{r}) \ . \end{aligned} \tag{32}$$

Here V_{ij} denotes V_{ud} for $\pi(\rho)$ and V_{us} for $K^{(*)}$, and $r \equiv \frac{M_{\Lambda_c}}{M_{\Lambda_b}}$, $t_{P(V)} \equiv \frac{m_{P(V)}}{M_{\Lambda_b}}$ where $m_{P(V)}$ is the mass of the pseudoscalar (vector) meson. And $y = \frac{1+r^2-t^2_{P(V)}}{2r}$. a_1 is the QCD coefficient which is taken as a free parameter in the discussion of nonleptonic decays [18,19].

The above expressions are spin averaged results. If we take into account the spin effects, the spin up-down asymmetry of Λ_b is a good parameter to analyze $\Lambda_b \to \Lambda_c$ nonleptonic decay. In this case, the decay rates are [3,20]

$$\Gamma(P) \propto 1 + \alpha(P)(\mathbf{S}_{\Lambda_c} + \mathbf{S}_{\Lambda_b}) \cdot \hat{\mathbf{p}}_{\Lambda_c} \ , \tag{33}$$

where

$$\alpha(P) = -\frac{2\kappa_P \mathrm{Re}(A^*B)}{|A|^2 + \kappa_P^2 |B|^2} \ , \tag{34}$$

and

$$\Gamma(V) \propto 1 + \alpha(V)\mathbf{S}_{\Lambda_b} \cdot \hat{\mathbf{p}}_{\Lambda_c} \ , \tag{35}$$

where

$$\alpha(V) = \frac{\kappa_V \mathrm{Re}(8t_V^2 F_1 G_1^* - CD^*)}{2[2t_V^2(\kappa_V^2|F_1|^2 + |G_1|^2) + \kappa_V^2|C|^2 + |D|^2]} \ . \tag{36}$$

Here $\mathbf{S}_{\Lambda_Q}$ is the spin vector of Λ_Q and $\hat{\mathbf{p}}_{\Lambda_c}$ is the unit vector of momentum of Λ_c. $\alpha_{P(V)}$ is the spin up-down asymmetry parameter of Λ_b.

The numerical results from QCD sum rule and large N_c limit are given in Table 3. Because y is near to 1.45, in the QCD sum rule case, we used the nonlinear fitting of the Isgur-Wise function $\xi(y) = (\frac{2}{y+1})^{0.5} \exp\left[-0.8\frac{y-1}{y+1}\right]$ [7]. The value of a_1 is taken to be 0.98 which is obtained from the decay $B \to D\pi$ [19]. Various decay constants are taken as follows; $f_\rho = 210$ MeV, $f_K = 158$ MeV and $f_{K^*} = 214$ MeV. The quark model results of Ref. [20] are also listed for comparison.

V. SUMMARY

We have analyzed the $\Lambda_b \to \Lambda_c$ semileptonic decay in the framework of HQET to the order of $1/m_c$ and $1/m_b$. The predictions for the Isgur-Wise functions and mass parameters

from QCD sum rule and large N_c method are used. In the large N_c limit, we argued that the subleading Isgur-Wise function vanishes. The decay rates, distributions and varuous asymmetry parameters are calculated numerically. Some of the Λ_b nonleptonic decays are also calculated. The numerical results can be checked by the experiments in the near future.

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FIGURE CAPTIONS

Fig 1. y distribution of the decay rates for (a) QCD sum rule and (b) large N_c limit. $\frac{d\Gamma_{T_\pm}}{dy}$ and $\frac{d\Gamma_{L_\pm}}{dy}$ are abbreviated as $T_\pm$ and $L_\pm$, respectively.

Fig 2. Comparison of the two models in the helicity componential y distribution of the decay rates: (a) $\frac{d\Gamma_{T_+}}{dy}$ (b) $\frac{d\Gamma_{T_-}}{dy}$ (c) $\frac{d\Gamma_{L_+}}{dy}$ (d) $\frac{d\Gamma_{L_-}}{dy}$ (e) $\frac{d\Gamma}{dy} = \frac{d\Gamma_{T_+}}{dy} + \frac{d\Gamma_{T_-}}{dy} + \frac{d\Gamma_{L_+}}{dy} + \frac{d\Gamma_{L_-}}{dy}$.

Fig 3. Lepton energy distribution of the decay rates for (a) QCD sum rule and (b) large N_c limit.

Fig 4. Comparison of the two models in the helicity componential lepton energy distribution of the decay rates: (a) $\frac{d\Gamma_{T_+}}{dE}$ (b) $\frac{d\Gamma_{T_-}}{dE}$ (c) $\frac{d\Gamma_{L_+}}{dE}$ (d) $\frac{d\Gamma_{L_-}}{dE}$ (e) $\frac{d\Gamma}{dE} = \frac{d\Gamma_{T_+}}{dE} + \frac{d\Gamma_{T_-}}{dE} + \frac{d\Gamma_{L_+}}{dE} + \frac{d\Gamma_{L_-}}{dE}$.

TABLE CAPTIONS

Table 1. The partial decay rates(in 10^{-14} GeV).

Table 2. Asymmetry parameters.

Table 3. Numerical results for Λ_b two-body nonleptonic decays. The QCD coefficient a_1 is taken to be 0.98 [19].

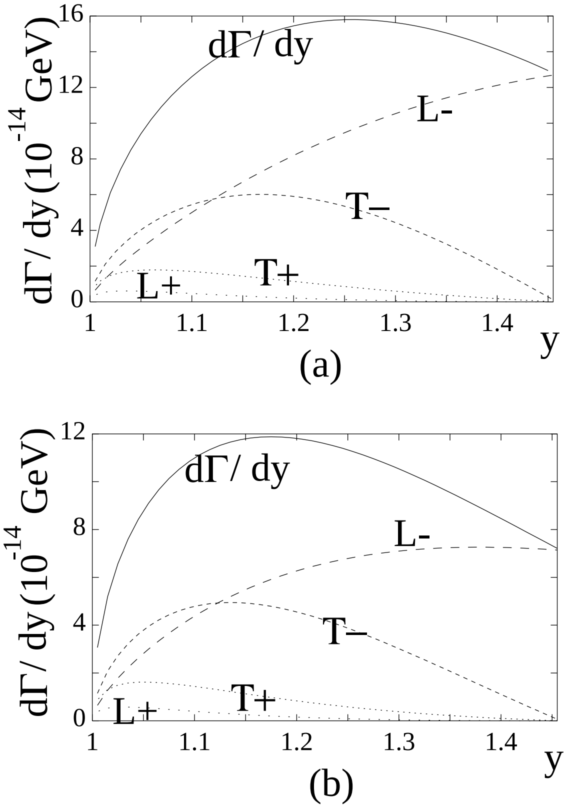
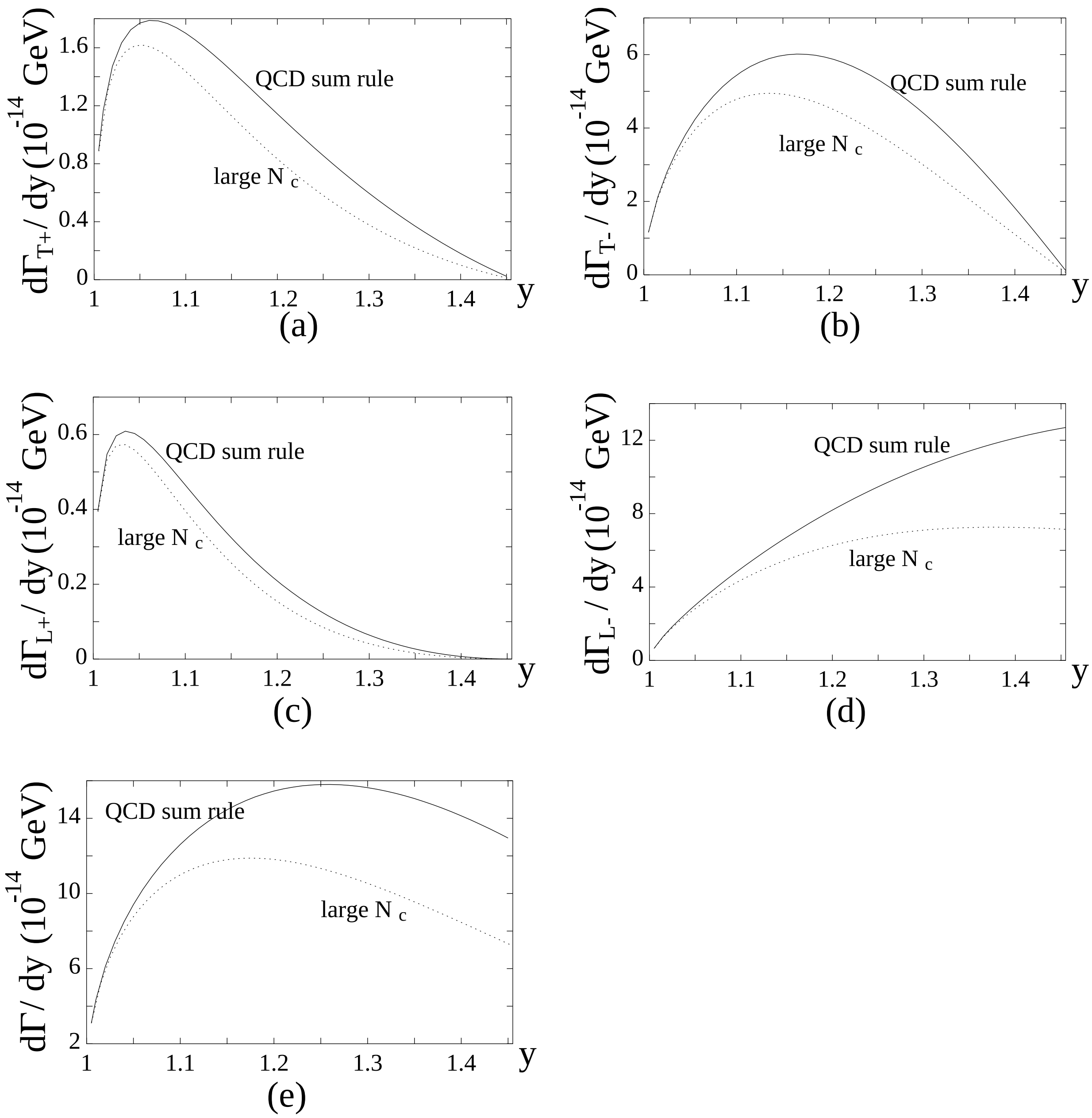

dΓ/ dy
L-
T−
T+
L+
dΓ/ dy (10^-14 GeV)
16
12
8
4
0
1
1.1
1.2
1.3
1.4
y
(a)
dΓ/ dy
L-
T−
T+
L+
dΓ/ dy (10^-14 GeV)
12
8
4
0
1
1.1
1.2
1.3
1.4
y
(b)

FIG. 1.



dΓT+/dy (10^-14 GeV)
QCD sum rule
large N c
y
(a)
dΓT- / dy (10^-14 GeV)
QCD sum rule
large N c
y
(b)
dΓL+/ dy (10^-14 GeV)
QCD sum rule
large N c
y
(c)
dΓL- / dy (10^-14 GeV)
QCD sum rule
large N c
y
(d)
dΓ/ dy (10^-14 GeV)
QCD sum rule
large N c
y
(e)

FIG. 2.

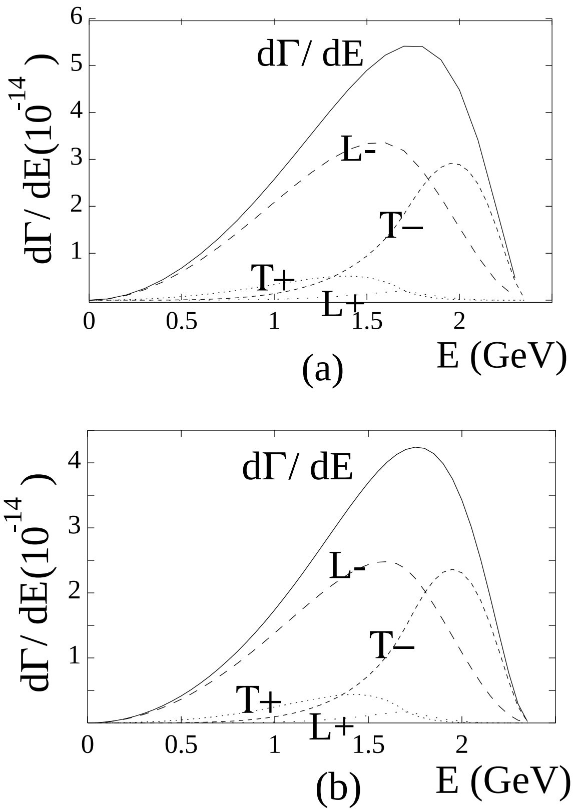
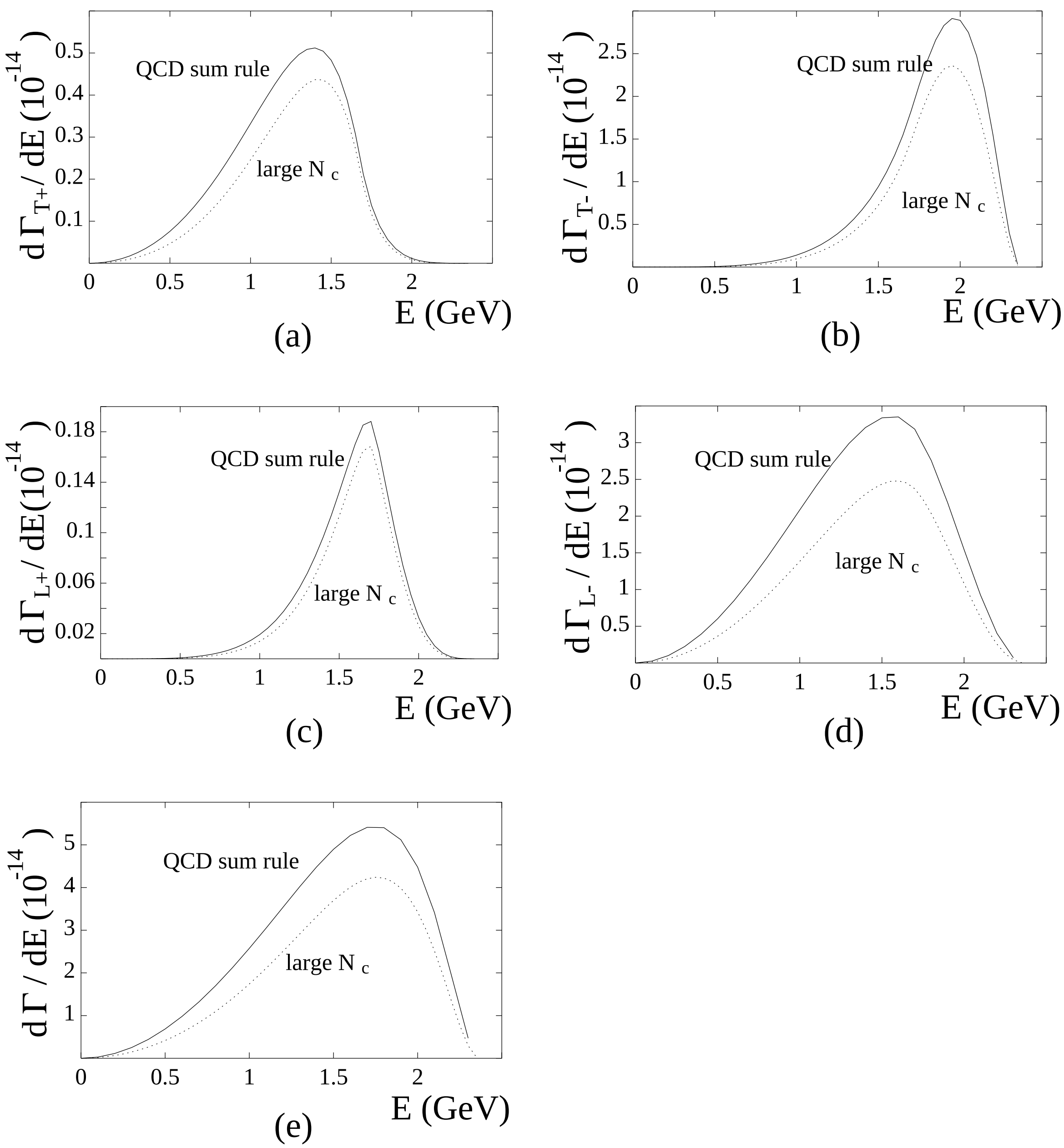


FIG. 3.



QCD sum rule
large N c
d Γ T+ / dE (10 -14)
E (GeV)
(a)
QCD sum rule
large N c
d Γ T- / dE (10 -14)
E (GeV)
(b)
QCD sum rule
large N c
d Γ L+ / dE(10 -14)
E (GeV)
(c)
QCD sum rule
large N c
d Γ L- / dE (10 -14)
E (GeV)
(d)
QCD sum rule
large N c
d Γ / dE (10 -14)
E (GeV)
(e)

FIG. 4.

TABLES

	Γ_{T+}	Γ_{T-}	Γ_{L+}	Γ_{L-}	Γ_{tot}
QCD sum rule	0.432	1.86	0.103	3.77	6.17
large N_c	0.343	1.45	0.087	2.63	4.51
quark model [17]	0.408	1.20	0.092	2.58	4.28

TABLE I.

	$<\alpha>$	$<\alpha'>$	$<\alpha''>$	$<\gamma>$	$<\alpha_P>$	$<\gamma_P>$
sum rule	-0.83	-0.14	-0.57	0.48	0.38	-0.17
large N_c	-0.81	-0.15	-0.53	0.50	0.34	-0.19
quark model [17]	-0.77	-0.11	-0.54	0.55	0.40	-0.16

TABLE II.

	$\Gamma(10^{10}\ \mathrm{sec}^{-1})$			α		
Modes	sum rule	large N_c	quark model [20]	sum rule	large N_c	quark model [20]
$\Lambda_c\pi$	0.780	0.406	0.342	-1.00	-1.00	-0.996
$\Lambda_c\rho$	1.08	0.583	0.489	-0.886	-0.885	-0.876
$\Lambda_c K$	0.057	0.030	0.025	-1.00	-1.00	-0.997
$\Lambda_c K^*$	0.055	0.030	0.025	-0.853	-0.853	-0.842

TABLE III.